Oceana High School

A MATH TOOLKIT FOR SCIENCE STUDENTS

I. Multiplying very big numbers or very small numbers:

a. Very big numbers

Suppose you want to multiply 2,455,600,000,000 x 7,981,400,000.

Step 1: Move the decimal point from the end of each number, place it in back of the first digit and count the number of places that the decimal point was moved.

2,455,600,000,000 → 2.455600000000 (12 places)

7,981,400,000 → 7.981400000 (9 places)

Step 2: Remove all the zeros to the right of the last non-zero digit and then multiply each number by 10 to the power of the number of places that the decimal point was moved.

2.455600000000 → 2.4556×10^{12}

7.981400000 → 7.981×10^{9}

Step 3: Multiply the two decimal numbers together on your calculator.

$$2.4556 \times 7.981 = 19.5981436$$

Step 4: Multiply the two exponents together in your head.

$$10^{12} \times 10^{9} = 10^{12+9} = 10^{21}$$

Step 5: Multiply the two numbers from steps 3 and 4.

$$19.5981436 \times 10^{21}$$

Step 6: If there are 2 digits in front of the decimal (as in this example), then move the decimal one more place to the left and add one to the exponent.

$19.5981436 \times 10^{21}$ → $1.95981436 \times 10^{22}$ (answer in scientific notation)

b. Very small numbers

Suppose you want to multiply 0.0000000000563 x 0.00000721

Step 1: Move the decimal to the right of the first non-zero digit and count the number of spaces that the decimal was moved.

0.0000000000563 → 5.63 (11 places)

0.00000721 → 7.21 (6 places)

Step 2: Multiply each number by 10 to the power of the number of places that the decimal was moved but this time put a minus sign in front of the exponent.

5.63×10^{-11} $\quad$ 7.21×10^{-6}

Step 3: Multiply the two decimal numbers together on your calculator.

$5.63 \times 7.21 = 40.5923$

Step 4: Multiply the two exponents together in your head.

$10^{-11} \times 10^{-6} = 10^{-11 + (-6)} = 10^{-17}$

Step 5: Multiply the two numbers from steps 3 and 4.

40.5923×10^{-17}

Step 6: If there are 2 digits in front of the decimal (as in this example), then move the decimal one more place to the left and add one to the exponent.

$40.5923 \times 10^{-17} \rightarrow 4.05923 \times 10^{-16}$ (answer in scientific notation)

c. A very big number times a very small number

Suppose you want to multiply 8,329,400,000,000,000 times 0.00000000913.

Step 1:	8,329,400,000,000,000	→	8.3294	(15 places)
	0.00000000913	→	9.13	(9 places)
Step 2:	8.3294×10^{15}			
	9.13×10^{-9}			(note that the exponent is negative)
Step 3:	$8.3294 \times 9.13 = 76.047422$			
Step 4:	$10^{15} \times 10^{-9} = 10^{15 + (-9)} = 10^{6}$			
Step 5:	76.047422×10^{6}			
Step 6:	7.6047422×10^{7}			(answer in scientific notation)

II. Dividing very big numbers or very small numbers:

a. Very big numbers

Suppose you want to divide 267,900,000,000 by 341,000,000

Step 1:	$\dfrac{2.679 \times 10^{11}}{3.41 \times 10^{8}}$	(convert both numbers to scientific notation)
Step 2:	$2.679 \div 3.41 = 0.78563$	(5 non-zero digits are enough to keep)
Step 3:	$10^{11} \div 10^{8} = 10^{11-8} = 10^{3}$	(Note that you **subtract** the exponents!)
Step 4:	0.78563×10^{3}	
Step 5:	7.8563×10^{2}	(scientific notation)

b. Very small numbers

Suppose you want to divide 0.000000000759 by 0.00000036

Step 1:	$\dfrac{7.59 \times 10^{-10}}{3.6 \times 10^{-7}}$	(convert to scientific notation)
Step 2:	$7.59 \div 3.6 = 2.1083$	(5 digits are kept)
Step 3:	$10^{-10} \div 10^{-7} = 10^{-10-(-7)} = 10^{-3}$	
Step 4:	2.1083×10^{-3}	

III. Combining multiplying and dividing:

Suppose you have to multiply and divide several numbers, like the following:

$$\frac{365{,}700{,}000 \times 0.00000062 \times 53{,}000}{799{,}000 \times 0.00000000000123}$$

Step 1: Convert all the numbers to scientific notation

$$\frac{(3.657 \times 10^{8}) \times (6.2 \times 10^{-7}) \times (5.3 \times 10^{4})}{(7.99 \times 10^{5}) \times (1.23 \times 10^{-12})}$$

Step 2: Gather all the decimal numbers together and use your calculator to get the answer

$$\frac{3.657 \times 6.2 \times 5.3}{7.99 \times 1.23} = 12.228 \qquad \text{(keep 5 digits)}$$

Step 3: Gather all the exponents together and get the answer in your head

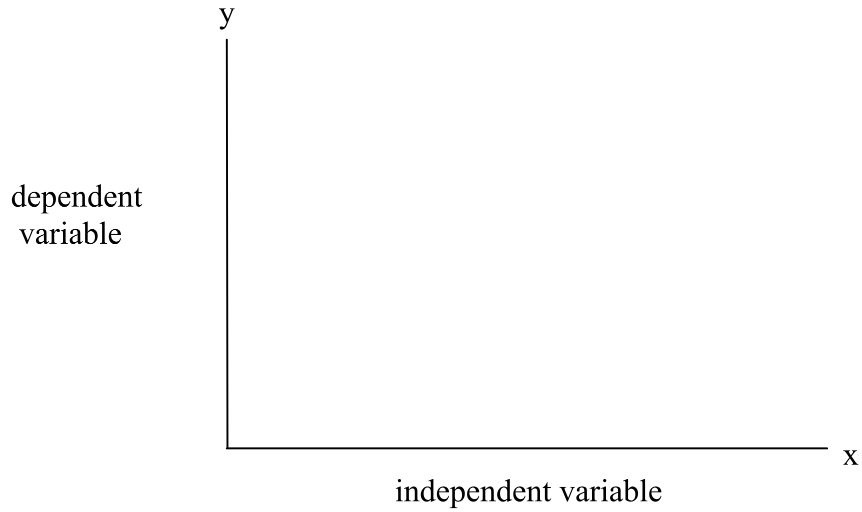
$$\frac{10^{8} \times 10^{-7} \times 10^{4}}{10^{5} \times 10^{-12}} = \frac{10^{8+(-7)+4}}{10^{5+(-12)}} = \frac{10^{5}}{10^{-7}} = 10^{5-(-7)} = 10^{12}$$

Step 4: Combine the answers from steps 2 and 3

$$12.228 \times 10^{12} = 1.2228 \times 10^{13}$$

IV. Graphing

a. Labelling the axes



A "variable" is any dimension (distance, mass, time, speed, etc.) that you are going to measure in an experiment.

An "independent" variable is one that you choose the values of <u>before</u> the experiment starts.

A "dependent" variable has values that you then measure in the experiment and that depend on whatever the value of the independent variable is.

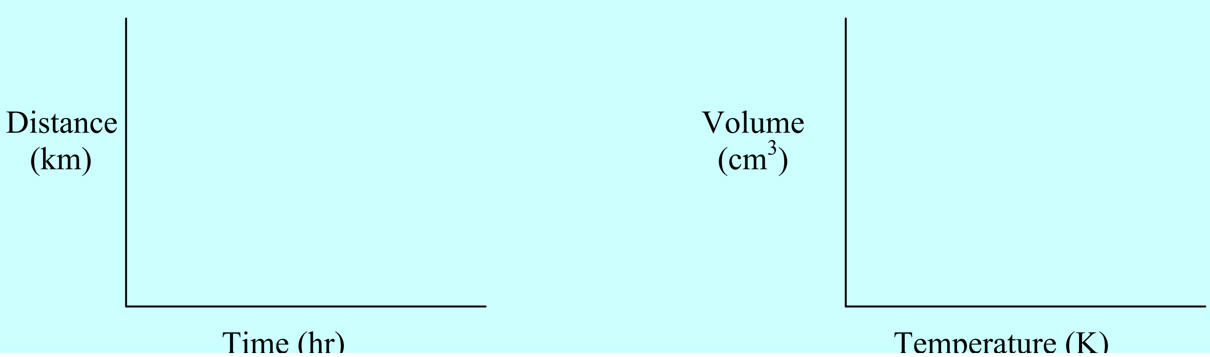
Example 1: Suppose you are driving along at 40 km/hr. How does the distance you travel depend on the time that you are traveling?

In this case, the two variables are ***time*** and ***distance.*** In order to know how far you travel, you would have to choose values for time (1 hr, 2 hr, 3hr, etc.) and then measure how far you traveled after each of those time periods. *Time* is therefore the independent variable, and *distance* is the dependent variable because how far you go depends on how long you travel.

Example 2: Balloons expand in volume when they are heated. Suppose you want to know how the volume of the balloon depends on its temperature.

In this case, the two variables are ***temperature*** and ***volume***. In order to know how the volume of the balloon changes, you would have to change the temperature and then measure the new volume of the balloon. *Temperature* is therefore the independent variable and *volume* is the dependent variable because the size of the balloon depends on what temperature it's at.

The graphs for examples 1 and 2 would be labeled as shown below:



b. Collecting experimental data in data tables and graphing the results

The simplest and best way to keep track of data that you collect in an experiment is to organize a data table **<u>first</u>** and, then, record the data in your table as you collect it. You should **<u>always</u>** record all measurements that you make in an experiment (raw data), and not just the calculations that you make from the measurements. For instance, in the second example above, you can't measure the volume of a balloon directly but you can measure its circumference. From that measurement you can then calculate the radius (from $c = 2\prod r$) and then volume ($V = 4/3\prod r^3$). You should **<u>not</u>** just record the volume calculation in your table, but also the circumference <u>measurements</u> and the radius calculations (Why?). Data tables and sample data for the two examples above might look as follows:

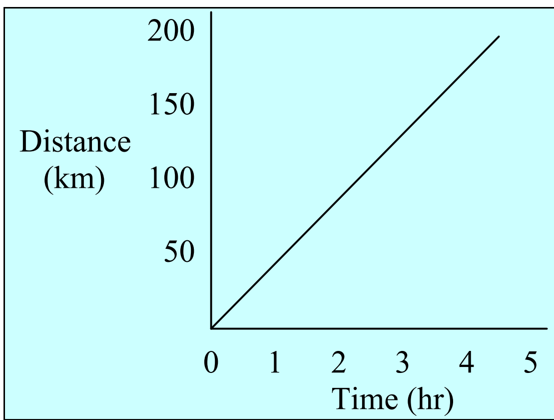
Example 1 –

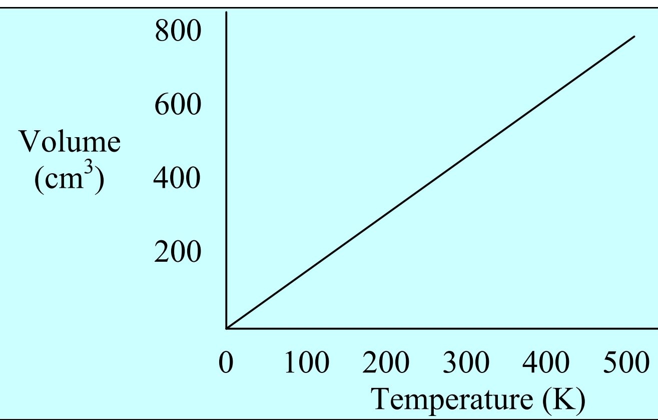
Interval	time (hr)	distance (km)
1	0	0
2	1	40
3	2	80
4	3	120
5	4	160

Example 2 –

Trial	temperature (K)	circumference (cm)	radius (cm)	volume (cm^3)
1	283	46.3	7.37	400
2	323	49.0	7.79	473
3	373	51.4	8.18	547
4	423	53.6	8.53	620
5	473	55.6	8.85	693

To graph these results, you make scales on the x-y axes that start at zero (the origin), are evenly spaced and where the farthest number is just larger than the largest number for each variable. In example 1, you could choose a scale of 0, 1, 2, 3, 4, 5 hrs for time and 0, 50, 100, 150, 200 km for distance. In example 2, you could choose 0, 100, 200, 300, 400 and 500K for the temperature scale and 0, 200, 400, 600 and 800 cm^3 for the volume scale. The graphs would then look as follows:



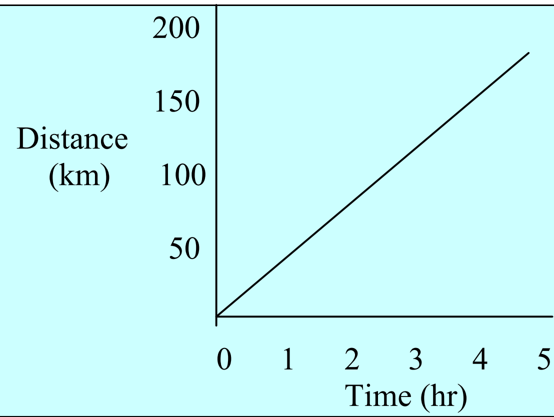


c. Error analysis

Are your measuring instruments perfect? Even if they were, would you be able to use them to make measurements perfectly? Why not? What this means is that any actual measurements that **<u>anyone</u>** makes will **<u>always</u>** have error due to imperfect instruments and human imperfection.

Take example 1 above, the table shows <u>calculated</u> data that I made up, not measurements from an actual experiment. The data is therefore perfect and gives a straight line on a graph in which all the data points are exactly on the line. This **<u>never</u>** happens in an actual experiment. Here is what a data table and graph of measurements from an actual experiment might look like:

Interval	time (hr)	distance (km)
1	0	0
2	1	38
3	2	83
4	3	116
5	4	165



Notice that the time measurements are accurate but the distance measurements are not. This is because time can be measured to the nearest hour with a stopwatch very accurately. Distance, however, depends on being able to keep a constant average velocity for an hour at a time which is not easy and therefore causes the distance traveled to be inaccurate. When this data is graphed, the points cannot be placed together on a straight line. This is called "scatter" – the points are scattered around a **best-fit** line. This scatter is **evidence** of error in the experiment and is what you should expect in a graph of <u>all</u> experiments!

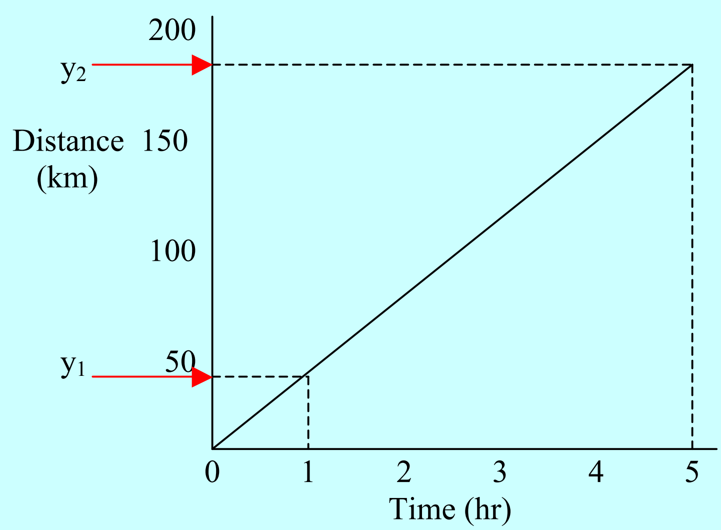
d. Calculating line slopes

The general formula for calculating the slope of a line on an x-y graph is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

To find the slope of a line, you pick two points that are **on the line** (x_1, y_1 and x_2, y_2). **Do not** use data points from your data table.

Take example 1 above, time is plotted on the x-axis and distance is plotted on the y-axis. To find the slope of the line you pick two values for x, let's say 1 hr (x_1) and 5 hr (x_2) and then you go to the graph to find the values for y as shown below:



The values taken from the graph are (approximately) y_1 = 42 km, and y_2 = 185 km. So the slope of the line is

$$m = (185\text{ km} - 42\text{ km})/(5\text{ hr} - 1\text{ hr}) = 35.75\text{ km/hr}$$

V. Using formulas and equations

There's no getting away from it. Science uses many formulas and equations, so, if you have math fear, get over it. Your best move is to learn how to use them as quickly as you can. Fortunately, most formulas in science have the same general form, namely

$$y = mx$$

Recognize this? It's from algebra I. There are two variables (x and y) and a constant (m). This is a **directly proportional** relationship. That means that as the value of x changes, the value of y will change by the same proportion. Notice also that this is the equation of a straight line with its starting point at the origin and has a graph just like the ones pictured above where m is the slope of the line. Here are some examples in chemistry and physics of formulas/equations that have this same form.

Chemistry		**Physics**	
density = mass/volume	(d = m/v)	force = mass x acceleration	(F = ma)
volume = constant x temperature	(V = cT)	distance = velocity x time	(d = vt)
volume = constant x # moles	(V = cn)	work = force x distance	(W = Fd)

How you solve problems using these equations is **always the same** – that's why you shouldn't be afraid! All you have to do is to be patient and to approach the problems stepwise.

Here are two examples – one from chemistry and one from physics. If you examine the solutions carefully, you will notice that the process of finding the solutions is exactly the same, even though the equations and concepts are different.

Example 1 – An unknown liquid has a density of 1.35 g/cm^3. A sample of this liquid occupies a volume of 250 cm^3. What is the mass of this sample?

Step 1: Identify the variables in the problem and write down their values.

density (d) = 1.35 g/cm^3
volume (v) = 250 cm^3
mass (m) = ? (notice that I also wrote down the variable that I am being asked to find)

Step 2: Write down the equation that includes all the variables that you wrote down.

$$d = m/v$$

Step 3: Rearrange the equation so that the thing you are being asked to find (m, in this case) is on the left side, and all the variables whose values are known (d and v) are on the right side.

$$m = dv$$

Step 4: Substitute into this equation the values you wrote down.

$$m = (1.35 \text{ g/cm}^3) \text{ x } (250 \text{ cm}^3)$$

Step 5: Do the math and cancel units as appropriate.

$$m = 337.5 \text{ g}$$

Example 2 – A car has a mass of 1,000 kg. Its engine pushes it with a force of 15,000 N. How fast does the car accelerate?

Step 1: Identify the variables in the problem and write down their values.

mass (m) = 1,000 kg
force (F) = 15,000 N = 15,000 kg · m/s^2
acceleration (a) = ? (notice that I also wrote down the variable that I am being asked to find)

Step 2: Write down the equation that includes all the variables that you wrote down.

$$F = ma$$

Step 3: Rearrange the equation so that the thing you are being asked to find (a, in this case) is on the left side, and all the variables whose values are known (m and F) are on the right side.

$$a = F/m$$

Step 4: Substitute into this equation the values that you wrote down.

$$a = (15{,}000 \text{ kg} \cdot \text{m/s}^2) / (1{,}000 \text{ kg})$$

Step 5: Do the math and cancel units, as appropriate.

$$a = 15 \text{ m/s}^2$$

Notice that in both examples, HOW I do the problem is the same. It does not matter whether it is chemistry or physics. You will save yourself a lot of time and grief if you use a methodical approach like I outlined here!

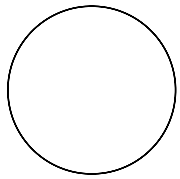
Inversely proportional relationships have the form

$$y = m/x$$

In this case, as x gets bigger, y gets smaller by the same proportion.

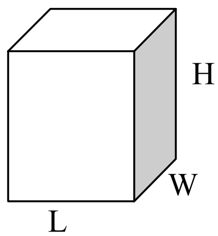
Formulas for areas and volumes of geometric shapes:
These are shapes that you commonly see in chemistry and physics.

Circles:



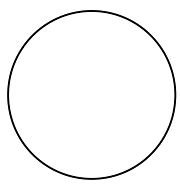
diameter = 2 x radius	$D = 2r$
circumference = 2 x pi x radius	$C = 2\Pi r$
area = pi x radius squared	$A = \Pi r^2$

Rectangular Solids:



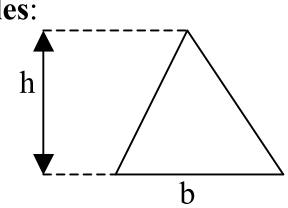
volume = length x width x height	$V = LWH$
surface area = 2 (length + width + height)	$A = 2(L + W + H)$

Spheres:



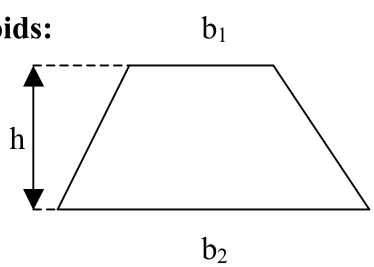
diameter = 2 x radius	$D = 2r$
circumference = 2 x pi x radius	$C = 2\Pi r$
surface area = 4 x pi x radius squared	$A = 4\Pi r^2$
volume = 4/3 x pi x radius cubed	$V = 4/3\ \Pi r^3$

Triangles:



Area = ½ x base x height	$A = 1/2bh$

Trapezoids:



Area = ½ x height x (base 1 + base 2)	$A = 1/2h\ (b_1 + b_2)$